Searching for the Optimal Ordering of Classes in Rule Induction

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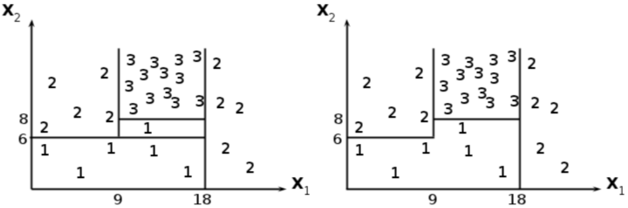
Outline

Rule Induction

- A rule set is typically an ordered list of rules.
- A rule contains a conjunction of terms and a class code.
- The terms are of the form $x_i = v$, $x_i < \theta$ or $x_i \geq \theta$.
- Example rule (Iris dataset)

  If $F_3 < 1.9$ **and** $F_4 \geq 5.1$ **Then** iris-setosa
  Else
      If $F_3 < 4.7$ **Then** iris-versicolor
      Else iris-virginica

Motivation



a) Current Approach' s Rule Set

If $(x_1<18)\&\&(x_2<6)$
Then Class = 1
If $(x_1>9)\&\&(x_2<8)$
Then Class = 1
If $(x_1<9)$
Then Class = 2
If $(x_1>18)$
Then Class = 2
Else
Class = 3

b) Another Ordering' s Rule Set

If $(x_1<9)\&\&(x_2>6)$
Then Class = 2
If $(x_1>18)$
Then Class = 2
If $(x_2< 8)$
Then Class = 1
Else
Class = 3

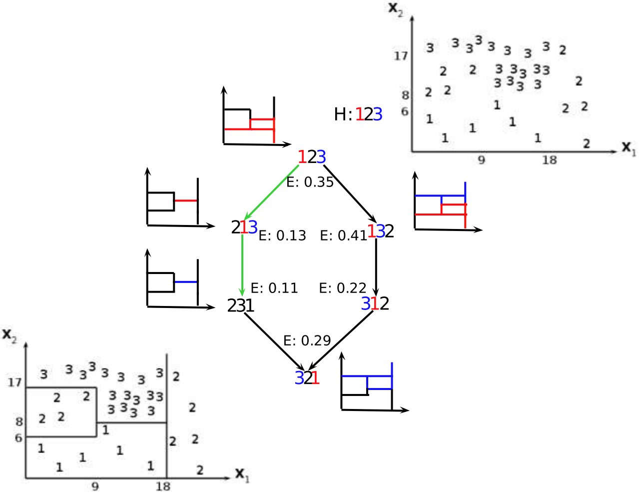
Ripper

```
RS = {}
C_i ordered in increasing prior probability
for p = 1 to K − 1
    Pos = C_p, Neg = C_{p+1}, ..., C_K
    RS_p = {}
    while D contains positive samples
        Divide D into Grow set G and Prune set P
        r = GrowRule(G)
        PruneRule(r, P)
        if CalculateError(r) > 0.5
            break
        else
            RS_p = RS_p + r
            Remove examples covered by r from D
    for i = 1 to 2
        OptimizeRuleset(RS_p, D)
        SimplifyRuleset(RS_p, D)
    RS = RS + RS_p
return RS
```

Forward Ordering Search: Algorithm

- Views optimizing the ordering of classes in Ripper as a search in the state space of all possible orderings.
- Forward search algorithm starts from an initial state (heuristic ordering) and use exchange operators to generate candidate states (orderings).
- Let say we have the ordering $C_1C_2C_3\ldots C_{K-1}C_K$. The exchange operator creates the following $K-1$ candidate orderings: $C_2C_1C_3\ldots C_{K-1}C_K$, $C_1C_3C_2\ldots C_{K-1}C_K$, $C_1C_2C_4\ldots C_{K-1}C_K, \ldots, C_1C_2C_3\ldots C_KC_{K-1}$.
- Candidates are evaluated via 10×10-fold cross-validation.
- We stop the search when no candidate improves on the current best.

Forward Ordering Search: Example

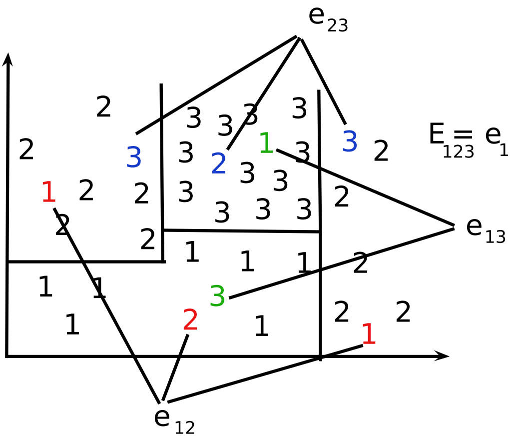


Pairwise Error Approximation: Assumption

The expected error of the Ripper trained with the ordering π_i, is the sum of $K(K-1)/2$ pairwise expected errors of classes:

$$\hat{E}_{\pi_i} = \sum_{j=1}^{K} \sum_{k=1,[j]<_{\pi_i}[k]}^{K} e_{jk}$$

Pairwise Error Approximation: Assumption



$$E_{123} = e_{12} + e_{13} + e_{23}$$

Pairwise Error Approximation: Algorithm

- Run Ripper algorithm N times with N random orderings π_i and get the test errors E_{π_i}.
- Minimize the total estimation error

$$E_t = \sum_{i=1}^{N} (E_{\pi_i} - \hat{E}_{\pi_i})^2$$

- Solve the following system of linear equations

$$\forall_{j,k} \frac{\partial E_t}{\partial e_{jk}} = 0$$

- Solved e_{jk}'s, search all possible class orderings to get the optimal ordering.

Datasets

Dataset	# of attributes	# of classes	Sample size
led7	7	7	3200
ocr	256	10	600
*opt*digits	64	10	3823
*pen*digits	16	10	7494
*seg*ment	19	7	2310
*shu*ttle	9	7	58000
*wine*quality	11	7	6497
yeast	8	10	1484

Results: Error rates

Dataset	Ripper	FOS	PEA
led7	31.83 ± 0.23	$\mathbf{29.67 \pm 0.23}$	$\mathbf{28.93 \pm 0.21}$
ocr	26.61 ± 0.58	$\mathbf{24.73 \pm 0.59}$	$\mathbf{22.08 \pm 0.54}$
opt	10.96 ± 0.14	$\mathbf{10.55 \pm 0.15}$	$\mathbf{8.57 \pm 0.12}$
pen	5.32 ± 0.07	$\mathbf{4.87 \pm 0.08}$	$\mathbf{4.49 \pm 0.08}$
seg	6.54 ± 0.17	$\mathbf{4.38 \pm 0.12}$	$\mathbf{5.03 \pm 0.14}$
shu	0.04 ± 0.00	$\mathbf{0.03 \pm 0.00}$	$\mathbf{0.01 \pm 0.00}$
wine	46.32 ± 0.16	46.23 ± 0.13	56.64 ± 0.23
yea	43.09 ± 0.35	$\mathbf{42.39 \pm 0.40}$	43.38 ± 0.36

Results: Complexity of FOS

Dataset	# of Orderings
led7	40
ocr	18
opt	25
pen	41
seg	22
shu	12
wine	17
yea	25

Results: Average Estimation Error of PEA

Dataset	$\overline{E_t}$
led7	0.31
ocr	0.63
opt	0.20
pen	0.08
seg	0.24
shu	1.86
wine	3.41
yea	2.67

Summary

- Current heuristic approach that orders the classes in a dataset according to their sample sizes, usually does not give the most accurate classification.
- FOS starts with the ordering the heuristic provides and searches for better orderings by swapping consecutive classes.
- PEA transforms the ordering search problem into an optimization problem and uses the solution of the optimization problem to extract the optimal ordering.