CSE 312 Final

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I. Question (15 points)

Solve the following recurrence relations and give a Θ bound for each of them.

- $T(n) = 2T(n/3) + 1$
- $T(n) = 8T(n/2) + n^3$
- $T(n) = T(n-1) + 2$

II. Question (15 points)

A student needs to take a certain number of courses to graduate, and these courses have prerequisites that must be followed. Assume that all courses are offered every semester and that the student can take an unlimited number of courses. Given a list of courses and their preprequisites, compute an order of courses which does not violate the prerequisite relation.

III. Question (15 points)

The object of Kevin Bacon game is to link a movie actor to Kevin Bacon via shared movie roles. The minimum number of links is an actor's Bacon number. For instance, Tom Hanks has a Bacon number of 1; he was in Apollo 13 with Kevin Bacon. Sally Fields has a Bacon number of 2, because she was in Forrest Gump with Tom Hanks, who was in Apollo 13 with Kevin Bacon. Almost all well-known actors have a Bacon number of 1 or 2. Assume that you have a comprehensive list of actors, with roles, and do the following:

- Give the algorithm to find an actor's Bacon number.
- Give the algorithm to find the actor with the highest Bacon number.

IV. Question (15 points)

The input is a list of league game scores (and there are no ties). If all teams have at least one win and a loss, we can generally "prove", by a silly transitivity argument, that any team is better than any other. For instance, in the six-team league where everyone plays three games, suppose we have the following results: A beat B and C; B beat C and F; C beat D; D beat E; E beat A; F beat D and E. Then we can prove that A is better than F, because A beat B, who in turn, beat F. Similarly, we can prove that F is better than A because F beat E and E beat A. Given a list of game scores and two teams X and Y, give an algorithm to find a proof that X is better than Y, or indicate that no proof of this form can be found.

V. Question (15 points)

A file contains only colons, spaces, newlines, commas, and digits in the following frequency: colon (100), space(605), newline(100), comma(705), 0(431), 1(242), 2(176), 3(59), 4(185), 5(250), 6(174), 7(199), 8(205), 9(217). Construct the Huffman code.

VI. Question (15 points)

Consider the following linear program.

$$
\begin{aligned}
maximize\ 5x + 3y & \\
5x - 2y &\geq 0 \\
x + 7 &\leq 7 \\
x &\leq 5 \\
x &\geq 0 \\
y &\geq 0
\end{aligned}
$$

Plot the feasible region and identify the optimal solution.

VII. Question (15 points)

A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below.

- Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue $1,000 per cubic meter.
- Material 2 has density 1 tons/cubic meter, maximum available amount 30 cubic meters, and revenue $1,200 per cubic meter.
- Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue $12,000 per cubic meter.

Write a linear program that optimizes revenue within the constraints.

VIII. Question (15 points)

Consider two teams, A and B, playing a series of games until one of the teams wins n games. Assume that the probability of A winning a game is the same for each game and equal to p, and the probability of A losing a game is q = 1 - p (Hence, there are no ties). Let P(i, j) be the probability of A winning the series if A needs i more games to win the series and B needs j more games to win the series.

- Set up a recurrence relation for P(i, j) that can be used by a dynamic programming algorithm.
- Write a pseudocode of the dynamic programming algorithm for solving this problem.