CSE 312 Midterm II (Exam in Class)

Olcay Taner YILDIZ

I. Algorithm Complexity (20 points)

The sum of the harmonic series 1, 1/2, 1/3, 1/4, . . ., diverges; that is

$$\sum_{i=1}^{\infty} \frac{1}{i} = \infty \tag{1}$$

Show that $\log n$ is both an upper and lower bound for the sum

$$\sum_{i=1}^{\infty} \frac{1}{i} \tag{2}$$

(Hint: To show an upper bound, decrease each denominator to the next power of two. For a lower bound, increase each denominator to the next power of 2.)

II. Divide and Conquer (20 points)

Suppose you are choosing between the following three algorithms:

- Algorithm A solves problems by dividing them into five subproblems of half the size, recursively solving each subproblem, and then combining the solutions in linear time.
- Algorithm B solves problems of size n by recursively solving two subproblems of size $n - 1$ and then combining the solutions in constant time.
- Algorithm C solves problems of size n by dividing them into nine subproblems of size $n/3$, recursively solving each subproblem, and then combining the solutions in O(n^2) time.

What are the running times of each of these algorithms (in big-O notation), and which would you choose?

III. Decomposition of Graphs (20 points)

In an undirected graph, the degree d(u) of a vertex u is the number of neighbors u has, or equivalently, the number of edges incident upon it.

(a) Show that in an undirected graph, $\sum_{u \in V} d(u) = 2|E|$.

(b) Use part (a) to show that in an undirected graph, there must be an even number of vertices whose degree is odd.